

J.K. SHAH CLASSES

MATHEMATICS & STATISTICS

FYJC TERMINAL TEST - 01

DURATION - 2 HR

SOLUTION SET

MARKS - 50

Q1. (A) Attempt ANY THREE OF THE FOLLOWING

(09)

01. $3x + y = 2$; $kx + 2y = 3$; $2x - y = -3$ are consistent. Find k

Q-1A

SOLUTION

$$3x + y = 2$$

$$kx + 2y = 3$$

$$2x - y = -3 \quad \text{are consistent}$$

Hence

$$\begin{vmatrix} + & - & + \\ 3 & 1 & 2 \\ k & 2 & 3 \\ 2 & -1 & -3 \end{vmatrix} = 0$$

$$3(-6 + 3) - 1(-3k - 6) + 2(-k - 4) = 0$$

$$3(-3) + 3k + 6 - 2k - 8 = 0$$

$$-9 + k - 2 = 0$$

$$-11 + k = 0$$

$$k = 11$$

02. If A.M. of two numbers exceeds their G.M. by 15 and their H.M. by 27, find the numbers

SOLUTION

$$A - G = 15 \quad \therefore G = A - 15 \dots (1)$$

$$A - H = 27 \quad \therefore H = A - 27 \dots (2)$$

$$G^2 = AH$$

$$(A - 15)^2 = A(A - 27)$$

$$A^2 - 30A + 225 = A^2 - 27A$$

$$225 = 3A$$

$$75 = A$$

SUBS IN (1)

$$G = 75 - 15 = 60$$

NOW ;

$$\begin{array}{l|l} A = 75 & G = 60 \\ \frac{a+b}{2} = 75 & G^2 = 3600 \\ a+b = 150 & ab = 3600 \\ \dots (3) & \dots (4) \end{array}$$

Solving

$$a(150 - a) = 3600$$

$$150a - a^2 = 3600$$

$$a^2 - 150a + 3600 = 0$$

$$a^2 - 120a - 30a + 3600 = 0$$

$$a(a - 120) - 30(a - 120) = 0$$

$$(a - 30)(a - 120) = 0$$

$$a = 30 \quad ; \quad a = 120$$

subs in (3)

$$b = 120 \quad ; \quad b = 30 \quad \text{ans : } 30, 120$$

03. in G.P. $t_4 = 24$; $t_9 = 768$. Find S_8

SOLUTION

$$t_4 = 24 \quad \therefore ar^3 = 24 \quad \dots (1)$$

$$t_9 = 972 \quad \therefore ar^8 = 768 \quad \dots (2)$$

$$\frac{ar^8}{ar^3} = \frac{768}{24}$$

$$r^5 = 32$$

$$r = 2$$

subs in (1)

$$a(2)^3 = 24$$

$$a.8 = 24$$

$$a = 3$$

Now

$$a = 3 ; r = 2 ; n = 8$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_8 = \frac{3(2^8 - 1)}{2 - 1}$$

$$= 3(256 - 1)$$

$$= 3(255)$$

$$= 765$$

04. If for a sequence , $S_n = 2n^2 + 5n$, find t_n and show that the sequence is an A.P.

SOLUTION

$$\begin{aligned}t_n &= S_n - S_{n-1} \\&= 2n^2 + 5n - [2(n-1)^2 + 5(n-1)] \\&= 2n^2 + 5n - [2(n^2 - 2n + 1) + 5n - 5] \\&= 2n^2 + 5n - [2n^2 - 4n + 2 + 5n - 5] \\&= 2n^2 + 5n - [2n^2 + n - 3] \\&= 2n^2 + 5n - 2n^2 - n + 3 \\&= 4n + 3\end{aligned}$$

$$\begin{aligned}t_{n-1} &= 4(n-1) + 3 \\&= 4n - 4 + 3 \\&= 4n - 1\end{aligned}$$

$$\begin{aligned}t_n - t_{n-1} &= 4n + 3 - (4n - 1) \\&= 4n + 3 - 4n + 1 \\&= 4 = \text{constant} \quad \text{Hence AP}\end{aligned}$$

01. Prove without expanding as far as possible

$$\begin{vmatrix} a^2 + 2a & 2a + 1 & 1 \\ 2a + 1 & a + 2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a - 1)^3$$

Q-1B

SOLUTION

$R_1 - R_2, R_2 - R_3$

$$= \begin{vmatrix} a^2 - 1 & a - 1 & 0 \\ 2a - 2 & a - 1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$

Taking $(a - 1)$ common from R_1 & R_2

$$= (a - 1)^2 \begin{vmatrix} a + 1 & 1 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$

$R_1 - R_2$

$$= (a - 1)^2 \begin{vmatrix} a - 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$

Taking $(a - 1)$ common from R_1

$$= (a - 1)^3 \begin{vmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$

Expanding the determinant

$$= (a - 1)^3 [1(1 - 0)]$$

$$= (a - 1)^3$$

02. Prove without expansion

$$\begin{vmatrix} x + y & y + z & z + x \\ z + x & x + y & y + z \\ y + z & z + x & x + y \end{vmatrix} = 2 \begin{vmatrix} x & y & z \\ z & x & y \\ y & z & x \end{vmatrix}$$

LHS

$C_1 - C_2$

$$= \begin{vmatrix} x - z & y + z & z + x \\ z - y & x + y & y + z \\ y - x & z + x & x + y \end{vmatrix}$$

$$C_1 + C_3$$

$$= \begin{vmatrix} 2x & y+z & z+x \\ 2z & x+y & y+z \\ 2y & z+x & x+y \end{vmatrix}$$

Taking '2' common from C_1

$$= 2 \begin{vmatrix} x & y+z & z+x \\ z & x+y & y+z \\ y & z+x & x+y \end{vmatrix}$$

$$C_3 - C_1$$

$$= 2 \begin{vmatrix} x & y+z & z \\ z & x+y & y \\ y & z+x & x \end{vmatrix}$$

$$C_2 - C_3$$

$$= 2 \begin{vmatrix} x & y & z \\ z & x & y \\ y & z & x \end{vmatrix} = \text{RHS}$$

Q2. (A) Attempt ANY TWO OF THE FOLLOWING

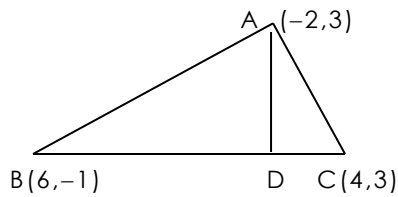
(06)

01. Find the coordinates of the orthocenter of a triangle whose vertices are
 $(-2,3)$, $(6,-1)$, $(4,3)$

Q-2A

SOLUTION

ALTITUDE AD



$$m_{BC} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{3 + 1}{4 - 6} = \frac{4}{-2} = -2$$

AD

$$m = \frac{1}{2} \text{ (AD } \perp \text{ BC) , A(-2,3)}$$

$$y - y_1 = m(x - x_1)$$

$$y - 3 = \frac{1}{2} (x + 2)$$

$$2y - 6 = x + 2$$

$$x - 2y = -8$$

$$y - 3 = 2(x - 4)$$

$$y - 3 = 2x - 8$$

$$2x - y = 5$$

ORTHOCENTER 'H'

$$x - 2y = -8$$

$$2x - y = 5 \quad \times 2$$

$$x - 2y = -8$$

$$4x - 2y = 10$$

$$\begin{array}{r} - \\ + \\ \hline -3x = -18 \end{array}$$

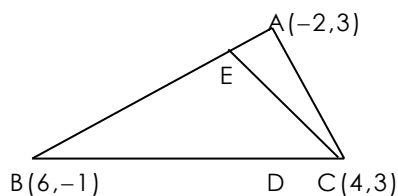
$$x = 6$$

subs in (1)

$$y = 7$$

H (6,7)

ALTITUDE CE



$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{3 + 1}{-2 - 6} = \frac{4}{-8} = -\frac{1}{2}$$

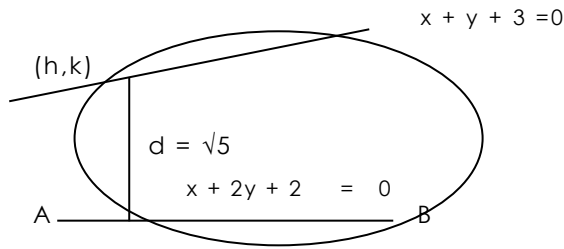
CE

$$m = 2 \text{ (CE } \perp \text{ AB) , C(4,3)}$$

$$y - y_1 = m(x - x_1)$$

02. Find points on the line $x + y + 3 = 0$ whose distance from $x + 2y + 2 = 0$ is $\sqrt{5}$ units

SOLUTION



STEP 1 :

Since (h, k) lies on $x + y + 3 = 0$, it must satisfy the equation
 $\therefore h + k = -3 \dots\dots (1)$

STEP 2 :

$$d = \sqrt{5}$$

$$\left| \frac{h + 2k + 2}{\sqrt{1^2 + 2^2}} \right| = \sqrt{5}$$

$$\frac{h + 2k + 2}{\sqrt{5}} = \pm \sqrt{5}$$

$$h + 2k + 2 = \pm 5$$

$$h + 2k + 2 = 5 \quad \left| \quad h + 2k + 2 = -5 \right.$$

$$h + 2k = 3 \dots\dots (2) \quad \left| \quad h + 2k = -7 \dots\dots (3) \right.$$

Solving (1) & (2)

$$\begin{array}{r} h + k = -3 \\ - \quad h + 2k = 3 \\ \hline -k = -6 \\ k = 6 \\ h + 6 = -3 \\ h = -9 \\ (-9, 6) \end{array}$$

Solving (1) & (3)

$$\begin{array}{r} h + k = -3 \\ - \quad h + 2k = -7 \\ \hline -k = 4 \\ k = -4 \\ h - 4 = -3 \\ h = 1 \\ (1, -4) \end{array}$$

03. if the acute angle between the lines $4x - y + 7 = 0$ and $kx - 5y - 9 = 0$ is 45° , find k

SOLUTION

$$4x - y + 7 = 0 ; \quad m_1 = -\frac{a}{b} = -\frac{4}{-1} = 4$$

$$kx - 5y - 9 = 0 ; \quad m_2 = -\frac{a}{b} = -\frac{k}{-5} = \frac{k}{5}$$

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 \cdot m_2} \right|$$

$$\tan 45 = \left| \frac{4 - \frac{k}{5}}{1 + \frac{4k}{5}} \right|$$

$$1 = \left| \frac{20 - k}{5 + 4k} \right|$$

$$\frac{20 - k}{5 + 4k} = 1$$

$$20 - k = 5 + 4k$$

$$15 = 5k$$

$$k = 3$$

$$\frac{20 - k}{5 + 4k} = -1$$

$$20 - k = -5 - 4k$$

$$3k = -25$$

$$k = -25/3$$

01. if $\cos\theta = -\frac{3}{5}$; $\pi < \theta < \frac{3\pi}{2}$. find $\frac{\operatorname{cosec}\theta + \cot\theta}{\sec\theta - \tan\theta}$

SOLUTION

θ lies in the III Quadrant

$\therefore \tan\theta$ & $\cot\theta$ are positive

$\cos\theta = -\frac{3}{5} ; \sec\theta = -\frac{5}{3}$

Q-2B

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta + \frac{9}{25} = 1$$

$$\sin^2\theta = 1 - \frac{9}{25}$$

$$\sin^2\theta = \frac{16}{25}$$

$\sin\theta = -\frac{4}{5} ; \operatorname{cosec}\theta = -\frac{5}{4}$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$= \frac{-4/5}{3/5}$$

$\tan\theta = \frac{4}{3} ; \cot\theta = \frac{3}{4}$

NOW
$$\frac{\operatorname{cosec}\theta + \cot\theta}{\sec\theta - \tan\theta}$$

$$= \frac{-\frac{5}{4} + \frac{3}{4}}{-\frac{5}{3} - \frac{4}{3}}$$

$$= \frac{-\frac{2}{4}}{-\frac{9}{3}} = \frac{1}{2} = \frac{1}{6}$$

02. Prove : $\sin A \cdot \sin(B - C) + \sin B \cdot \sin(C - A) + \sin C \cdot \sin(A - B) = 0$

LHS

$$\begin{aligned} & \sin A \cdot \sin(B - C) \\ &= \sin A \cdot (\sin B \cos C - \cos B \sin C) \\ &= \sin A \cdot \sin B \cdot \cos C - \sin A \cdot \cos B \cdot \sin C \end{aligned}$$

$$\begin{aligned} & \sin B \cdot \sin(C - A) \\ &= \sin B \cdot (\sin C \cos A - \cos C \sin A) \\ &= \sin B \cdot \sin C \cdot \cos A - \sin B \cdot \cos C \cdot \sin A \\ &= \cos A \cdot \sin B \cdot \sin C - \sin A \cdot \sin B \cdot \cos C \end{aligned}$$

$$\begin{aligned} & \sin C \cdot \sin(A - B) \\ &= \sin C \cdot (\sin A \cos B - \cos A \sin B) \\ &= \sin C \cdot \sin A \cdot \cos B - \sin C \cdot \cos A \cdot \sin B \\ &= \sin A \cdot \cos B \cdot \sin C - \cos A \cdot \sin B \cdot \sin C \end{aligned}$$

$$\begin{aligned} \text{LHS} &= \sin A \cdot \sin(B - C) + \sin B \cdot \sin(C - A) + \sin C \cdot \sin(A - B) \\ &= \sin A \cdot \sin B \cdot \cos C - \sin A \cdot \cos B \cdot \sin C + \cos A \cdot \sin B \cdot \sin C - \sin A \cdot \sin B \cdot \cos C \\ &\quad + \sin A \cdot \cos B \cdot \sin C - \cos A \cdot \sin B \cdot \sin C \\ &= 0 \end{aligned}$$

03. Prove : $\tan 54^\circ = \tan 36^\circ + 2 \tan 18^\circ$

SOLUTION

$$\tan 54^\circ = \tan (36 + 18)$$

$$\tan 54^\circ = \frac{\tan 36^\circ + \tan 18^\circ}{1 - \tan 36^\circ \cdot \tan 18^\circ}$$

$$\tan 54 - \tan 54 \cdot \tan 36 \cdot \tan 18 = \tan 36 + \tan 18$$

$$\tan 54 - \tan(90-36) \cdot \tan 36 \cdot \tan 18 = \tan 36 + \tan 18$$

$$\tan 54 - \cot 36 \cdot \tan 36 \cdot \tan 18 = \tan 36 + \tan 18$$

$$\tan 54^\circ - \tan 18^\circ = \tan 36^\circ + \tan 18^\circ$$

$$\tan 54^\circ = \tan 36^\circ + 2 \tan 18^\circ.$$

01. 7 persons sit in a row . Find the total number of seating arrangements if
- 3 persons A , B ,C sit together in particular order
 - A , B and C sit together in any order
 - A and B occupy the end seats

Q-3A

SOLUTION

- a) 3 persons A , B ,C sit together in particular order

Consider A , B , C as 1 set

1 set of A , B , C & 4 others can be arranged in ${}^5P_5 = 5!$ ways

Having done that ;

Since A , B , C sit together in a particular order , they cannot be further arranged

By fundamental principle of Multiplication

$$\text{Total arrangements} = 5! = 120$$

- b) A , B and C sit together in any order

Consider A , B , C as 1 set

1 set of A , B , C & 4 others can be arranged in ${}^5P_5 = 5!$ ways

Having done that ;

A , B , C can then be arranged among themselves in ${}^3P_3 = 3!$ ways

By fundamental principle of Multiplication

$$\begin{aligned} \text{Total arrangements} &= 5! \times 3! \\ &= 120 \times 6 = 720 \end{aligned}$$

- c) A and B occupy the end seats

A and B can be arranged onto the end seats in ${}^2P_2 = 2!$ ways

Having done that ;

5 others can then be arranged among themselves in ${}^5P_5 = 5!$ ways

By fundamental principle of Multiplication

$$\begin{aligned} \text{Total arrangements} &= 2! \times 5! \\ &= 2 \times 120 = 240 \end{aligned}$$

02. how many even numbers of four digits can be formed using digits 0 , 1 , 2 , 3 & 4 if no digit being used more than once

SOLUTION

Step 1 : 4 – digit numbers formed

Thousand place can be filled by any one of the 4 digits ('0' excluded) in 4 ways

Having done that , remaining 3 places can be filled by any 3 of remaining 4 digits in 4P_3 ways

By fundamental principle of Multipliation ,

$$\text{Total Numbers formed} = 4 \times {}^4P_3 = 4 \times 4 \times 3 \times 2 = 96$$

Step 2 : Odd Numbers

Unit place can be filled by any one of digits 1 , 3 in 2P_1 ways

Having done that ,

Thousand place can be filled by any one the remaining 3 digits ('0' excluded) in 3P_1 ways

Having done that , remaining 2 places can be filled by any 2 of remaining 3 digits in 3P_2 ways

By fundamental principle of Multipliation ,

$$\text{No. of Odd Numbers formed} = {}^2P_1 \times {}^3P_1 \times {}^3P_2 = 2 \times 3 \times 3 \times 2 = 36$$

Hence

$$\underline{\text{No. of Even numbers formed}} = 96 - 36 = 60$$

(B) Attempt ANY THREE OF THE FOLLOWING

(09)

01. From the following data , find the percentage of workers who are weighing more than 68 kgs

Weight (in kg):	50 – 55	55 – 60	60 – 65	65 – 70	70 – 75
No. of worker:	15	18	30	25	12

Q-3B

SOLUTION :

CI	f	cf
50 – 55	15	15
55 – 60	18	33
60 – 65	30	63
65 – 70	25	88 ←
70 – 75	12	100

let nth worker weigh 68 kg. This worker is in class 65 – 70

$$68 = 65 + \frac{n - 63}{25} (70 - 65)$$

$$3 = \frac{n - 63}{25} \cdot (5)$$

$$3 = \frac{n - 63}{5}$$

$$15 = n - 63 \quad \therefore n = 78$$

78th worker weighs 68 kgs

$$\therefore \text{No. of workers weighing more than 68kgs} = 100 - 78 = 22$$

$$\therefore \text{Percentage of workers weighing more than 68kgs} = \frac{22}{100} \times 100 = 22\%$$

02. Monthly Bal. less than	1000	900	800	700	600	500	400	300	200
No of A/c holder	500	498	480	475	440	374	300	125	25

Find the 7th decile

SOLUTION :

CI	f	cf
100 – 200	25	25
200 – 300	100	125
300 – 400	175	300
400 – 500	74	374
500 – 600	66	440
600 – 700	35	475
700 – 800	5	480
800 – 900	18	498
900 – 1000	2	500

$$d_7 = 7 \frac{N}{10} = \frac{7(500)}{10} = 350$$

$$D_7 = L_1 + \frac{d_7 - c}{f} (L_2 - L_1)$$

$$= 400 + \frac{350 - 300}{74} (500 - 400)$$

$$= 400 + \frac{50}{74} (100)$$

$$= 400 + 0.675 (100)$$

$$= 467.5 \text{ (Rs)}$$

03. Solve : $\log_3 x + \log_9 x + \log_{243} x = \frac{34}{5}$

SOLUTION

$$\frac{\log x}{\log 3} + \frac{\log x}{\log 9} + \frac{\log x}{\log 243} = \frac{34}{5}$$

$$\frac{\log x}{\log 3} + \frac{\log x}{\log 3^2} + \frac{\log x}{\log 3^5} = \frac{34}{5}$$

$$\frac{\log x}{\log 3} + \frac{\log x}{2 \log 3} + \frac{\log x}{5 \log 3} = \frac{34}{5}$$

$$\left(\frac{\log x}{\log 3} \cdot \frac{1}{2} + \frac{1}{2} \right) + \frac{1}{5} = \frac{34}{5}$$

$$\frac{\log x}{\log 3} \left(\frac{10 + 5 + 2}{10} \right) = \frac{34}{5}$$

$$\frac{\log x}{\log 3} \left(\frac{17}{10} \right) = \frac{34}{5}$$

$$\frac{\log x}{\log 3} = 4$$

$$\log x = 4 \log 3$$

$$\log x = \log 3^4$$

$$x = 81$$

04. if $a^2 - 12ab + 4b^2 = 0$; Prove $\log(a + 2b) = \frac{1}{2}(\log a + \log b) + 2\log 2$

SOLUTION :

$$a^2 - 12ab + 4b^2 = 0$$

$$a^2 + 4b^2 = 12ab$$

ADDING '+4ab' ON BOTH SIDES

$$a^2 + 4ab + b^2 = 12ab + 4ab$$

$$(a + 2b)^2 = 16ab$$

INSERTING LOG ON BOTH SIDES

$$\log (a + 2b)^2 = \log 16ab$$

$$2\log (a + 2b) = \log 16 + \log a + \log b$$

$$2\log (a + 2b) = \log 2^4 + \log a + \log b$$

$$2\log (a + 2b) = 4\log 2 + \log a + \log b$$

$$\log (a + 2b) = \frac{4\log 2 + \log a + \log b}{2}$$

$$\log (a + 2b) = \frac{4\log 2}{2} + \frac{1}{2}(\log a + \log b)$$

$$\log (a + 2b) = 2\log 2 + \frac{1}{2}(\log a + \log b)$$

..... PROVED

Q4. (A) Attempt ANY **TWO OF THE FOLLOWING**

01. Find coefficient of variation for the following data : 10 ; 20 ; 18 ; 12 ; 15

STEP 1 :

x	x - \bar{x}	(x - \bar{x}) ²
10	- 5	25
20	5	25
18	3	9
12	-3	9
15	0	0
75		68

$$\bar{x} = \frac{\sum x}{n} = \frac{75}{5} = 15$$

STEP 2 :

$$\begin{aligned} \sigma &= \sqrt{\frac{\sum(x - \bar{x})^2}{n}} \\ &= \sqrt{\frac{68}{5}} = \sqrt{13.6} \end{aligned}$$

taking log on both sides

$$\begin{aligned} \log \sigma &= \frac{1}{2}(\log 13.6) \\ &= \frac{1}{2}(1.1335) \\ &= \frac{1.1335}{2} \end{aligned}$$

$$\log \sigma = 0.5668$$

$$\begin{aligned} \sigma &= AL(0.5668) \\ &= 3.689 \end{aligned}$$

STEP 3 :

$$\begin{aligned} CV &= \frac{\sigma}{\bar{x}} \times 100 \\ &= \frac{3.689}{15} \times 100 \\ &= \frac{368.9}{15} \\ &= 24.59\% \end{aligned}$$

02.

Bowley's coefficient of skewness is 0.6 .

The sum of upper and lower quartiles is 100 and the median is 38 . Find the upper and lower quartiles

$$Q_3 + Q_1 = 100 ; M = 38 ; SK_B = 0.6$$

$$SK_B = \frac{(Q_3 - Q_2) - (Q_2 - Q_1)}{(Q_3 - Q_2) + (Q_2 - Q_1)}$$

$$SK_B = \frac{Q_3 + Q_1 - 2M}{Q_3 - Q_1}$$

$$0.6 = \frac{100 - 2(38)}{Q_3 - Q_1}$$

$$Q_3 - Q_1 = \frac{100 - 76}{0.6}$$

$$Q_3 - Q_1 = \frac{24}{0.6} = \frac{240}{6} = 40$$

$$\text{Now } Q_3 + Q_1 = 100$$

$$Q_3 - Q_1 = 40$$

$$\hline 2 Q_3 = 140$$

$$Q_3 = 70$$

Subs in

$$Q_3 + Q_1 = 100$$

$$70 + Q_1 = 100$$

$$Q_1 = 30$$

03.

in a series of 5 observations , the value of mean and variance is 3 and 2 . If three observations are 1 , 3 & 5 find the remaining two

let the other 2 observations be a & b

$$\bar{x} = \frac{\sum x}{n}$$

$$3 = \frac{1 + 3 + 5 + a + b}{5}$$

$$15 = 9 + a + b$$

$$a + b = 6$$

$$\therefore b = 6 - a \dots\dots (1)$$

$$\sigma = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2}$$

$$\sigma^2 = \frac{\sum x^2}{n} - \bar{x}^2$$

$$2 = \frac{1 + 9 + 25 + a^2 + b^2}{5} - 3^2$$

$$2 = \frac{35 + a^2 + b^2}{5} - 9$$

$$11 = \frac{35 + a^2 + b^2}{5}$$

$$55 = 35 + a^2 + b^2$$

$$a^2 + b^2 = 20$$

$$a^2 + (6 - a)^2 = 20 \quad \text{from (1)}$$

$$a^2 + 36 - 12a + a^2 = 20$$

$$2a^2 - 12a + 16 = 0$$

$$a^2 - 6a + 8 = 0$$

$$a = 4 \qquad a = 2$$

$$b = 6 - a \qquad b = 6 - a$$

$$b = 2 \qquad b = 4$$

\(\therefore\) the other two observations are 2 & 4

(B) Attempt ANY TWO OF THE FOLLOWING

(06)

01. for moderately skewed distribution mean = 40 ; Karl Pearson's coefficient of skewness is 0.1 & coeff. of variation is 20% .

Find mode

STEP 1 : SD

$$CV = \frac{\sigma}{\bar{x}} \times 100$$

$$20 = \frac{\sigma}{40} \times 100$$

$$\sigma = 8$$

STEP 2 : MODE

$$Skp = \frac{\text{Mean} - \text{Mode}}{\sigma}$$

$$0.1 = \frac{40 - \text{Mode}}{8}$$

$$0.8 = 40 - \text{Mode}$$

$$\text{Mode} = 40 - 0.8$$

$$\text{Mode} = 39.2$$

STEP 3 : MEDIAN

$$\text{Mean} - \text{mode} = 3(\text{mean} - \text{median})$$

$$40 - 39.2 = 3(40 - \text{median})$$

$$0.8 = 3(40 - \text{median})$$

$$0.27 = 40 - \text{median}$$

$$\text{median} = 40 - 0.27 = 39.73$$

Q-4B

02. ${}^{11}P_{r-1} : {}^{12}P_{r-2} = 14 : 3$. Find r

SOLUTION

$$\frac{{}^{11}P_{r-1}}{{}^{12}P_{r-2}} = \frac{1}{14}$$

$$\frac{\frac{11!}{(11-r+1)!}}{\frac{12!}{(12-r+2)!}} = \frac{1}{14}$$

$$\frac{\frac{11!}{(12-r)!}}{\frac{12!}{(14-r)!}} = \frac{1}{14}$$

$$\frac{11!}{(12-r)!} \times \frac{(14-r)!}{12!} = \frac{1}{14}$$

$$\frac{11!}{12!} \times \frac{(14-r)!}{(12-r)!} = \frac{1}{14}$$

$$\frac{11! \times (14-r)(13-r)(12-r)!}{12 \cdot 11! (12-r)!} = \frac{1}{14}$$

$$\frac{1}{(n+3)(n-2)} = \frac{1}{14}$$

$$(n+3)(n-2) = 14$$

$$(n+3)(n-2) = 7 \cdot 2$$

On Comparing ;

$$n+3 = 7$$

$$n = 7 - 3 = 4$$

03. the first three moments about 4 are 1 , 4 and 10 respectively . Find the coefficient of skewness γ_1

$$A = 4 , \mu_1(a) = 1 , \mu_2(a) = 4 , \mu_3(a) = 10$$

$$\mu_1 = 0$$

$$\begin{aligned} \mu_2 &= \mu_2(a) - \mu_1(a)^2 \\ &= 4 - 1 \\ &= 3 \end{aligned}$$

$$\begin{aligned} \mu_3 &= \mu_3(a) - 3\mu_1(a)\mu_2(a) + 2\mu_1(a)^3 \\ &= 10 - 3(1)(4) + 2(1) \\ &= 10 - 12 + 2 \\ &= 0 \end{aligned}$$

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3}$$

$$= \frac{0}{27}$$

$$= 0$$

$$\gamma_1 = \sqrt{\beta_1} = 0$$

All The Best for next exam , don't miss it

Ashish Sir @ JKSC